while L^* is as in Eq. (21). Values for L^*, Y^*_{TURB} and Y^*_{LAM} as functions of u_∞ and Θ_w/T_∞ for CO, N_2 mixtures at 1 amagat density are shown in Fig. 1. Since the values of ν for CO and N_2 are nearly identical, the mixture ratio is unimportant. In Fig. 1, (Θ_w/T_∞) and ν have been evaluated using a reference temperature halfway between T_w and T_∞ to replace T_∞ for large $(T_w - T_\infty)$. Regions in x, u_∞, and T_∞ where the flow is turbulent have been delineated using the criteria $Re_x = 2 \times 10^5$ or $Gr_x = 10^9$.

Conclusion

Approximate expressions for the effect on optical path length through a turbulent vertical boundary layer caused by the combined presence of forced and free convection have been obtained to first order in the asymptotic cases of dominant forced convection and dominant free convection. The effect in both cases is a reduction of the boundary-layer thickness. Characteristic scaling lengths have been presented which aid in the optical analysis of the flowfield.

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Cathode Geometry and Spoke Mode Operation of MPD Accelerators

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Introduction

SEVERAL authors have reported azimuthal nonuniformities, or rotating spokes, in the arc region and exhaust plume of MPD accelerators.[1-6] The amplitude and frequency of these rotating disturbances have been shown to be dependent on the propellant gas molecular weight, mass flow rate, applied current, and applied magnetic field strength.[1,2,5] Larson[3] and Allario, Jarrett and Hess[4] have shown the presence of a critical magnetic field strength at which transition from uniform to spoke-like discharge occurs. In addition, several investigators have attempted to provide theoretical bases for the observed spoke phenomena.[7-9]

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All of these studies have presented correlations of spoke mode operation with the accelerator operating parameters. As Cochran and Fay[5] have pointed out, no systematic study of the effect of electrode geometry on the occurence of the spoke mode has been made to date. However, this Note presents experimental results which suggest that the electrode geometry may have a significant effect on the mode of accelerator operation.

Experiment

An investigation of possible spoke mode operation was carried out for the plasma accelerator configuration shown in Fig. 1. The device was geometrically similar to that used by Larson[3] except for the cathode design, which was modified to provide increased cathode spot stability. The cathode was made from 0.64 cm o.d. thoriated tungsten rod, with a 0.32 cm i.d. hole, approximately 1.0 cm deep, drilled in the tip. This cavity provided stable cathode attachment for all operating conditions encountered in this investigation. In contrast, when a cone-tipped cathode was used in the otherwise identical accelerator the cathode attachment point moved erratically over the tip surface for applied magnetic field strengths below 0.05 tesla.

The effect of approximately 40 hours running time on the simple hollow cathode of this study is illustrated in Fig. 2. The erosion pattern indicates arc attachment within the cavity.

Since the luminosity of the rotating spoke has been shown to be significantly different from the rest of the arc region,[6] an optical technique was used to identify spoke mode operation. An image of a small portion of the arc region plasma located at a radial position midway between the anode

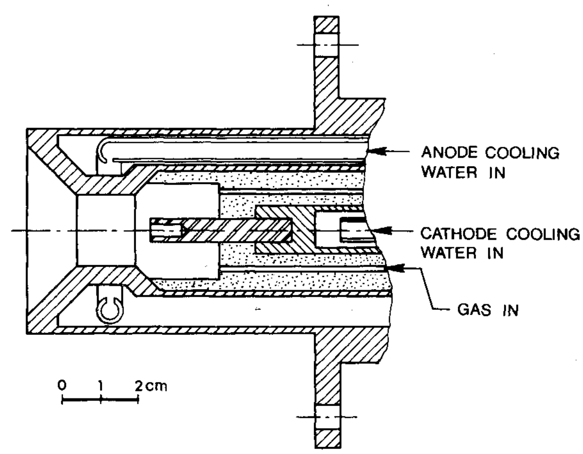

Fig. 1 Schematic cross-section of plasma accelerator.

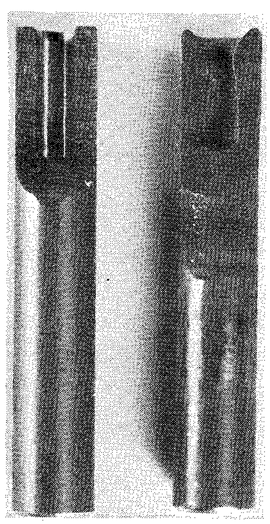

Fig. 2 Section view of new and used cathode inserts.

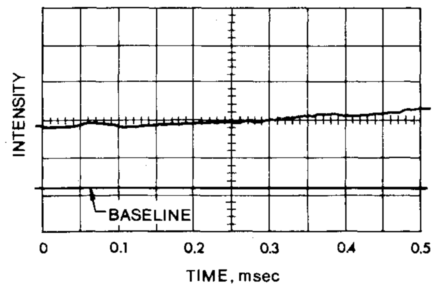

Fig. 3 Typical oscillogram, photomultiplier current vs time.

and the cathode was focussed on a photomultiplier. Periodic variation in the photomultiplier output signal would indicate the presence of a rotating spoke. Further discussion of the optical technique and equipment is given in Ref. 10.

This investigation was carried out for argon mass flow rates of 0.04-0.12 g/sec, exhaust tank pressures of 2.0×10^{-2}-7.5×10^{-2} torr, currents of 70-200 A, and magnetic field strengths of 0.0-0.24 tesla.

Results and Discussion

No periodic intensity oscillations, except those attributed to the 180 Hz power supply ripple, were observed for any combination of operating conditions. Specifically, no oscillations were present in the 25-300 kHz range expected for rotating spoke discharges. A copy of a typical oscillogram, showing luminous intensity as a function of time, is given in Fig. 3.

These results were somewhat surprising in light of the previous reports which predicted critical magnetic fields in the 0.1-0.2 tesla range, with spoke rotational frequencies on the order of 50 kHz for the reported operating conditions. The possibility of insufficient oscilloscope response was checked by replacing the initial 300 kHz bandwidth instrument with another oscilloscope having an upper frequency limit of 150 MHz. Possible inhibition of spoke operation by relatively large power supply ripple was also considered. A capacitive filter was inserted in the power supply output circuit to reduce the ripple by approximately one order of magnitude. The additional experiments confirmed the absence of periodic oscillations for the range of operating conditions considered.

The primary difference between the present experimental accelerator and those of the previously mentioned authors was in the cathode design. A consideration of the strong temperature dependence of electron emission and the thermal profile of the simple hollow cathode would suggest that this design would exhibit substantially more stable attachment than the more common conical design.

We suggest that the observed cathode spot stability may have had a direct effect on the discharge mode, maintaining a uniform discharge in an operating region where spoke mode operation has been observed by previous authors. This result merits further study since the discharge mode may have a direct effect on the electrode lifetime and the operating efficiency of a plasma accelerator.

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Turbulence-Model Predictions for a Flat Plate Boundary Layer

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Introduction

RESEARCH with the turbulence model equations of Saffman on a variety of incompressible and compressible flows has met with considerable success.[2-6] In this instance, the model equations were employed in a calculation of the downstream development of mean flow quantities in a flat plate, zero-pressure-gradient, incompressible turbulent boundary layer—a problem not considered previously.

A. The Turbulence Model Equations for an Incompressible, Zero-Pressure Gradient Boundary Layer

Using the boundary-layer approximation, the Reynolds shear stress is postulated to satisfy the following constitutive relation[1]

$$-\overline{uv} = A(e/\omega)(\partial U/\partial y) \tag{1}$$

where e is the turbulence kinetic energy, and ω is a pseudo-vorticity. The transport equations for e and ω, employing the boundary-layer approximation, are the following:

$$U\frac{\partial e}{\partial x} + V\frac{\partial e}{dy} = \alpha'' e\left|\frac{\partial U}{\partial y}\right| - e\omega + \frac{\partial}{\partial y}\left[\left(A'\frac{e}{\omega} + \nu\right)\frac{\partial e}{\partial y}\right] \tag{2}$$

$$U\frac{\partial \omega^2}{\partial x} + V\frac{\partial \omega^2}{\partial y} = \alpha'\omega^2\left|\frac{\partial U}{\partial y}\right| - \beta'\omega^3 + \frac{\partial}{\partial y}\left[\left(A''\frac{e}{\omega} + \nu\right)\frac{\partial \omega^2}{\partial y}\right] \tag{3}$$

where ν is the kinematic viscosity. The constants appearing in the previous equations are given the following values:[1] $A = 0.09$; $A' = A'' = 0.045$; $\beta' = 5/3$; $\alpha'' = 0.3$; $\alpha' = 0.163$. It

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